Introducing Extensions

Frege's Principle

A Farewell to Psychol-ogism

Extensions for Words and Phrases

Truth Values as Extensions of Sentences

ESSLLI Summerschool 2014: Intro to Compositional Semantics

Thomas Ede Zimmermann, Goethe-Universität Frankfurt
Wolfgang Sternefeld, Universität Tübingen

Second Lecture: Introducing Extensions

Introducing Extensions

Introducing Extensions
Frege's Principle
A Farewell to Psychologism
Extensions for Words and Phrases
Truth Values as Extensions of Sentences

Our plan for this course:

- Monday: Tuning in: Structural Ambiguity (Wolfgang)
- **Tuesday: Introducing Extensions (Ede)**
- Wednesday: Composing Extensions (Wolfgang)
- Thursday: Quantifiers (Wolfgang and Ede)
- Friday: Propositions and Intensions (Ede)

Frege's Principle

Introducing Extensions

Frege's Principle

A Farewell to Psychol-ogism

Extensions for Words and Phrases

Truth Values as Extensions of Sentences

Two arrangements of unambiguous words can different meanings:

(1) a. Fritz kommt
Fritz is-coming

b. Kommt Fritz
is-coming Fritz

Whereas the verb-second structure in (a) is normally interpreted as a declarative sentence, the verb-first structure in (b) is interpreted as a yes-no-question.

Frege's Principle

Introducing Extensions

Frege's Principle

A Farewell to Psychologism

Extensions for Words and Phrases

Truth Values as Extensions of Sentences

(2) *Frege's Principle of Compositionality*
The meaning of a composite expression is a function of the meaning of its immediate constituents and the way these constituents are put together.

A Farewell to Psychologism

Introducing Extensions

Frege's Principle

A Farewell to Psychol-ogism

Extensions for Words and Phrases

Truth Values as Extensions of Sentences

When learning a new word, we learn how to combine a certain pronunciation, its phonetics and phonology, with its meaning. Thereby, a previously meaningless sequence of sounds like *schmöll* becomes vivid, we associate with it the idea of someone who isn't thirsty any more. In this case, one might be tempted to say that the **meaning** of an expression is the idea or conception (*Vorstellung*) a speaker associates with its utterance.

A Farewell to Psychologism

Introducing Extensions

Frege's Principle

A Farewell to Psychol-ogism

Extensions for Words and Phrases

Truth Values as Extensions of Sentences

Schreiben, die bleiben Höhepunkte abendländischer Briefkultur, ausgewählt von Kaplan Klappstuhl, Folge 27.

An die Dudenredaktion, Abt. Neue Worte.

Betr. Anregung

Sehr geehrte Herren !

Mir ist aufgefallen, daß die deutsche Sprache ein Wort zuwenig hat. Wenn man nicht mehr " hungrig " ist, ist man "satt " .
Was ist man jedoch, wenn man nicht mehr "durstig" ist ? Na ? Naa ? Na bitte ! Dann "hat man seinen Durst gestillt" oder "man ist nicht mehr durstig" und was dergleichen unschöne Satzbandwürmer mehr sind . Ein k n a p p e s einsilbiges Wort für besagten Zustand fehlt jedoch, ich würde vorschlagen, dafür die Bezeichnung " schmöll " einzuführen und in Ihre Lexika auf - zunehmen .

Mit vorzüglicher Hoachtung

Werner Schmöll

A Farewell to Psychologism

Introducing Extensions

Frege's Principle

A Farewell to Psychol-ogism

Extensions for Words and Phrases

Truth Values as Extensions of Sentences

To the data editors of the *Duden* publishers, dept. new words

re: suggestion

Dear Sirs,

I have noticed that the German language lacks a word. If you are no longer hungry, you are full. But what are you if you are no longer thirsty? Eh? Then you have 'sated your thirst' or you are 'no longer thirsty' or some similarly inelegant circumlocution. But we have no short monosyllabic word for this condition. I would suggest that you introduce the term 'schmöll' and include it in your reference works.

Yours faithfully,
Werner Schmöll

A Farewell to Psychologism

Introducing Extensions

Frege's Principle

A Farewell to Psychol-ogism

Extensions for Words and Phrases

Truth Values as Extensions of Sentences

When learning a new word, we learn how to combine a certain pronunciation, its phonetics and phonology, with its meaning. Thereby, a previously meaningless sequence of sounds like *schmöll* becomes vivid, we associate with it the idea of someone who isn't thirsty any more. In this case, one might be tempted to say that the **meaning** of an expression is the idea or conception (*Vorstellung*) a speaker associates with its utterance.

Introducing Extensions

Frege's Principle

A Farewell to Psychol-ogism

Extensions for Words and Phrases

Truth Values as Extensions of Sentences

(Fregean and Wittgensteinian) ...

Introducing Extensions

Frege's Principle

A Farewell to Psychol-ogism

Extensions for Words and Phrases

Truth Values as Extensions of Sentences

(Fregean and Wittgensteinian) ...

(oops)

... objections ...

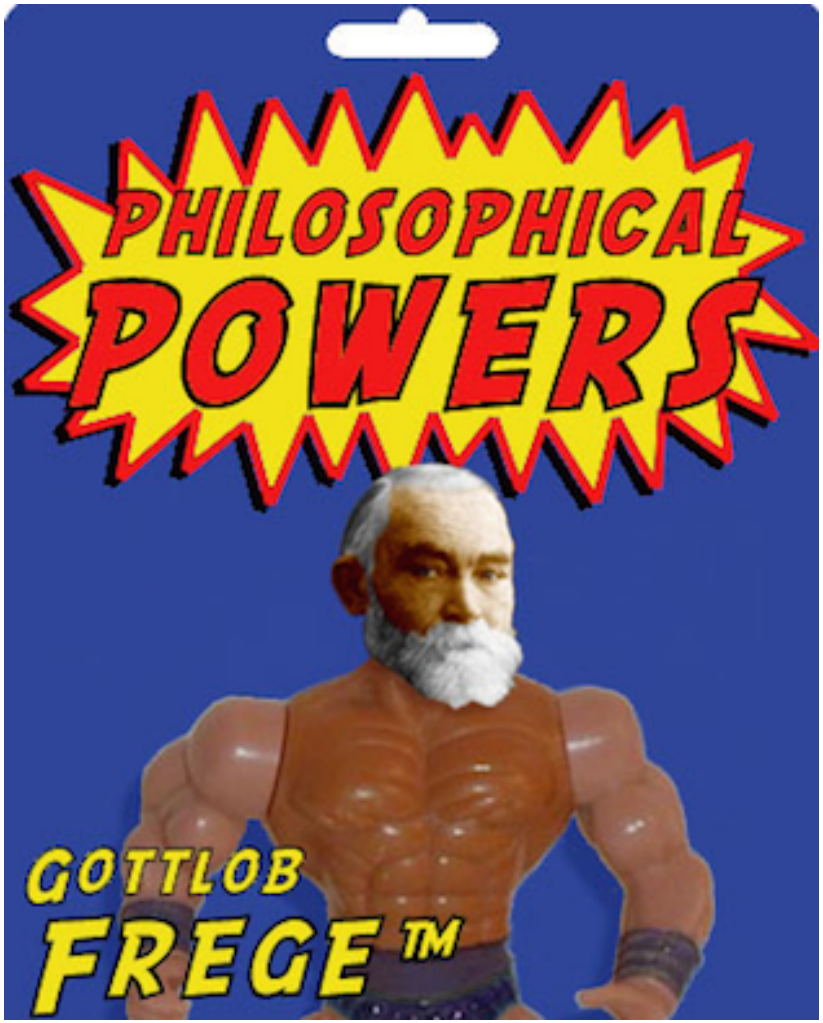
Introducing Extensions

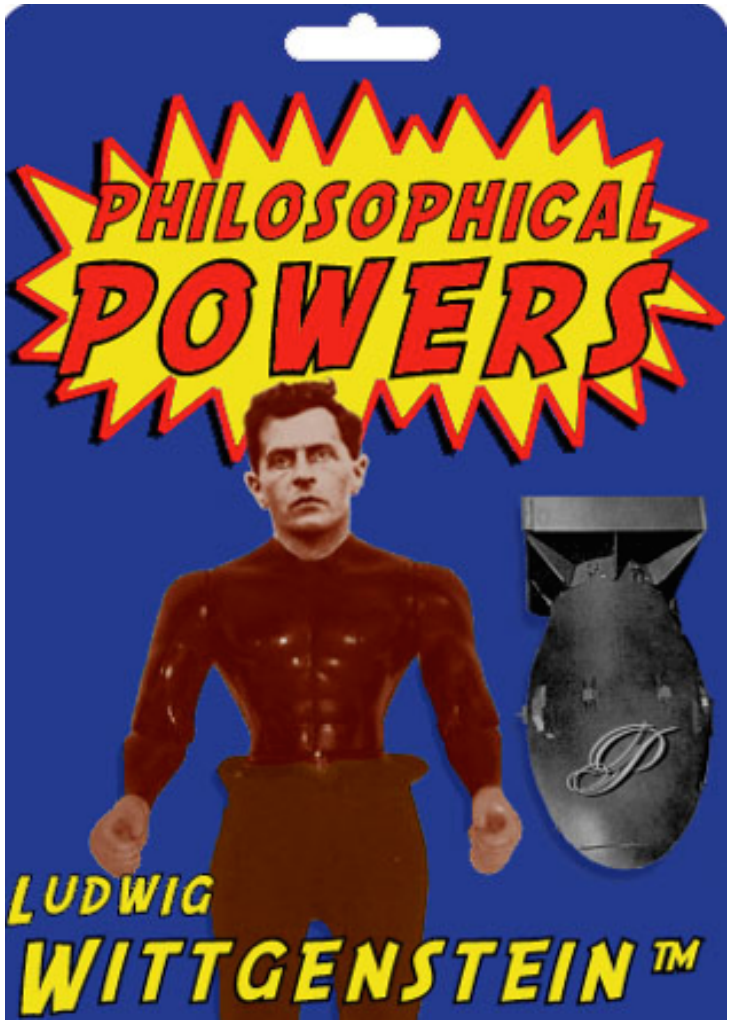
Frege's Principle

A Farewell to Psychol-ogism

Extensions for Words and Phrases

Truth Values as Extensions of Sentences





A Farewell to Psychologism

... against such a “**psychologistic**” notion of meaning:

Introducing Extensions

Frege’s Principle

A Farewell to Psychol-ogism

Extensions for Words and Phrases

Truth Values as Extensions of Sentences

A Farewell to Psychologism

... against such a “**psychologistic**” notion of meaning:

- **Subjectiveness**: Different speakers may associate different things with a single word at different occasions: such “meanings,” however, cannot be objective, but will rather be influenced by personal experience, and one might wonder how these “subjective meanings” serve communication between different subjects.

Introducing Extensions
Frege's Principle
A Farewell to Psychol-ogism
Extensions for Words and Phrases
Truth Values as Extensions of Sentences

A Farewell to Psychologism

Introducing Extensions

Frege's Principle

A Farewell to Psychol-ogism

Extensions for Words and Phrases

Truth Values as Extensions of Sentences

... against such a "**psychologistic**" notion of meaning:

- **Subjectiveness**: Different speakers may associate different things with a single word at different occasions: such "meanings," however, cannot be objective, but will rather be influenced by personal experience, and one might wonder how these "subjective meanings" serve communication between different subjects.
- **Limited Coverage**: We can have mental images of nouns like *horse* or *table*, but what on earth could be associated with words like *and, most, only, then, of, if*, . . . ?

A Farewell to Psychologism

Introducing Extensions

Frege's Principle

A Farewell to Psychol-ogism

Extensions for Words and Phrases

Truth Values as Extensions of Sentences

... against such a "**psychologistic**" notion of meaning:

- **Subjectiveness**: Different speakers may associate different things with a single word at different occasions: such "meanings," however, cannot be objective, but will rather be influenced by personal experience, and one might wonder how these "subjective meanings" serve communication between different subjects.
- **Limited Coverage**: We can have mental images of nouns like *horse* or *table*, but what on earth could be associated with words like *and, most, only, then, of, if*, . . . ?
- **Irrelevance**: Due to different personal experiences, speakers can have all sorts of associations without this having any influence on the meaning of an expression.

A Farewell to Psychologism

Introducing Extensions
Frege's Principle
A Farewell to Psychol-ogism
Extensions for Words and Phrases
Truth Values as Extensions of Sentences

... against such a "**psychologistic**" notion of meaning:

- **Subjectiveness**: Different speakers may associate different things with a single word at different occasions: such "meanings," however, cannot be objective, but will rather be influenced by personal experience, and one might wonder how these "subjective meanings" serve communication between different subjects.
- **Limited Coverage**: We can have mental images of nouns like *horse* or *table*, but what on earth could be associated with words like *and, most, only, then, of, if*, . . . ?
- **Irrelevance**: Due to different personal experiences, speakers can have all sorts of associations without this having any influence on the meaning of an expression.
- **Privacy**: The associations of an individual person are in principle inaccessible to other speakers. So, again, how can they be used for interpersonal communication?

A Farewell to Psychologism

Introducing Extensions

Frege's Principle

A Farewell to Psychol-ogism

Extensions for Words and Phrases

Truth Values as Extensions of Sentences

On the other hand ...

MEANING SERVES COMMUNICATION ... and so:

A Farewell to Psychologism

Introducing Extensions

Frege's Principle

A Farewell to Psychol-ogism

Extensions for Words and Phrases

Truth Values as Extensions of Sentences

On the other hand ...

MEANING SERVES COMMUNICATION ... and so:

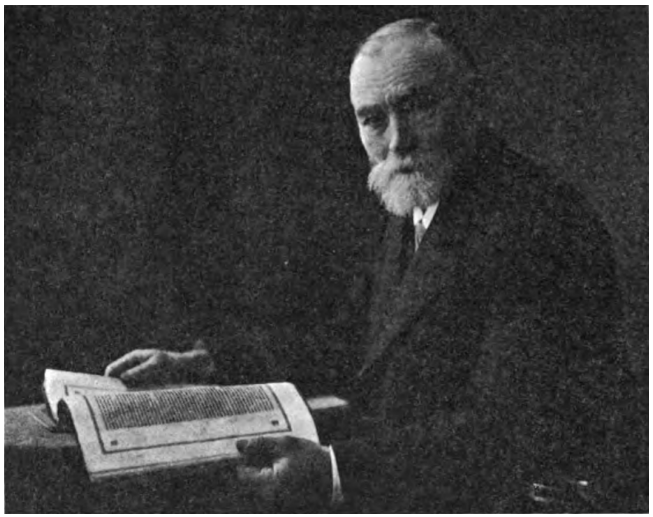
MEANINGS ought to be identified with

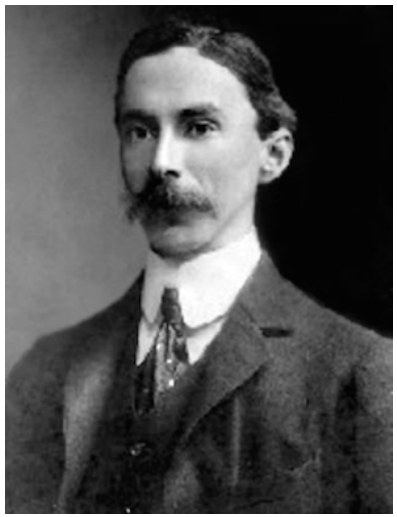
COMMUNICATIVE FUNCTIONS of expressions

... as in the tradition of ...

LOGICAL SEMANTICS

Introducing Extensions
Frege's Principle
A Farewell to Psychol-ogism
Extensions for Words and Phrases
Truth Values as Extensions of Sentences





Introducing Extensions

Frege's Principle

A Farewell to Psychol-ogism

Extensions for Words and Phrases

Truth Values as Extensions of Sentences

... or (more recently)
FORMAL SEMANTICS

Introducing Extensions

Frege's Principle

A Farewell to Psychol-ogism

Extensions for Words and Phrases

Truth Values as Extensions of Sentences

LOGICAL [or FORMAL] SEMANTICS

Meanings $\approx$ (certain) **communicative functions** of expressions, viz.:

- **Content**: *Which information* is expressed ...
- **Reference**: ... and *what* this information is *about*

Introducing Extensions
Frege's Principle
A Farewell to Psychol-ogism
Extensions for Words and Phrases
Truth Values as Extensions of Sentences

LOGICAL [or FORMAL] SEMANTICS

The **meaning** of any expressions has (at least) **two components**, viz. its:

- **intension** $\approx$ its contribution to the content of expressions in which it occurs
- **extension**: $\approx$ its contribution to the reference of expressions in which it occurs

In the simplest cases:

- Intension is content.
- Extension is reference.

We will start with the latter ...

Introducing Extensions
Frege's Principle
A Farewell to Psychologism
Extensions for Words and Phrases
Truth Values as Extensions of Sentences

Some examples:

(3) — Tübingen, Prof. Arnim v. Stechow (**proper names**)
— the president of the US (**definite descriptions**)
— table, horse, book (**nouns**)
— bald, red, stupid (**adjectives**)
— nobody, nothing, no dog (**negative quantifiers**)

Extensions for Words and Phrases

Introducing Extensions

Frege's Principle

A Farewell to Psychologism

Extensions for Words and Phrases

Truth Values as Extensions of Sentences

Some examples:

(3) — Tübingen, Prof. Arnim v. Stechow (**proper names**)
— the president of the US (**definite descriptions**)
— table, horse, book (**nouns**)
— bald, red, stupid (**adjectives**)
— nobody, nothing, no dog (**negative quantifiers**)

- What do these expressions refer to?

Introducing Extensions
Frege's Principle
A Farewell to Psychologism
Extensions for Words and Phrases
Truth Values as Extensions of Sentences

Some examples:

(3) — Tübingen, Prof. Arnim v. Stechow (**proper names**)
— the president of the US (**definite descriptions**)
— table, horse, book (**nouns**)
— bald, red, stupid (**adjectives**)
— nobody, nothing, no dog (**negative quantifiers**)

- What do these expressions refer to?
- What is their contribution to reference?

Extensions for Words and Phrases

Introducing Extensions

Frege's Principle

A Farewell to Psychol-ogism

Extensions for Words and Phrases

Truth Values as Extensions of Sentences

[What do these expressions refer to?]

Referential expressions like

- proper names (like *Stuttgart*, *Edward Snowden*, ...)
- definite descriptions (like *the capital of Baden-Württemberg*, *the whistle blower*...)
- (some) personal pronouns (like *she*)
- ...

(are used to) refer to persons, places, or other **individuals**.

Extensions for Words and Phrases

Introducing Extensions

Frege's Principle

A Farewell to Psychol-ogism

Extensions for Words and Phrases

Truth Values as Extensions of Sentences

[What do these expressions refer to?]

Referential expressions like

- proper names (like *Stuttgart*, *Edward Snowden*, ...)
- definite descriptions (like *the capital of Baden-Württemberg*, *the whistle blower*...)
- (some) personal pronouns (like *she*)
- ...

(are used to) refer to persons, places, or other **individuals**.

The referent of a referential expression also forms its **extension**.

Extensions for Words and Phrases

Introducing Extensions

Frege's Principle

A Farewell to Psychol-ogism

Extensions for Words and Phrases

Truth Values as Extensions of Sentences

[What do these expressions refer to?]

- **common (count) nouns** like *table*, *car*, ...

as well as some ('intersective')

- **adjectives** like *blond*, *rectangular*, ...

do not refer to single individuals but show **multiple** reference.

Extensions for Words and Phrases

Introducing Extensions
Frege's Principle
A Farewell to Psychologism
Extensions for Words and Phrases
Truth Values as Extensions of Sentences

[What do these expressions refer to?]

- **common (count) nouns** like *table*, *car*, ...

as well as some ('intersective')

- **adjectives** like *blond*, *rectangular*, ...

do not refer to single individuals but show **multiple** reference.

The set of all its referents forms the **extension** of such a multiply extensional expression.

Extensions for Words and Phrases

Introducing Extensions

Frege's Principle

A Farewell to Psychologism

Extensions for Words and Phrases

Truth Values as Extensions of Sentences

NB1: The extension of

- *current German chancellor*

is the set of all current German chancellors – i.e., a set with one member.

Extensions for Words and Phrases

Introducing Extensions

Frege's Principle

A Farewell to Psychol-ogism

Extensions for Words and Phrases

Truth Values as Extensions of Sentences

However, the extension of

- ***the*** *current German chancellor*

is the current German chancellor, i.e., a person.

Introducing Extensions

Frege's Principle

A Farewell to Psychol-ogism

Extensions for Words and Phrases

Truth Values as Extensions of Sentences

SO:

- *current German chancellor* (whose extension is {*A.M.*}),

and:

- *the current German chancellor*

do not have the same extension[1]!

[1] on standard set-theoretic assumptions

Introducing Extensions

Frege's Principle

A Farewell to Psychol-ogism

Extensions for Words and Phrases

Truth Values as Extensions of Sentences

NB2: The extension of

- *current French king*

is the set of all current French kings – i.e., the empty set.

Introducing Extensions

Frege's Principle

A Farewell to Psychol-ogism

Extensions for Words and Phrases

Truth Values as Extensions of Sentences

However, the extension of

- ***the*** *current king of France*

would have to be the current French king
... but there is no such (existing) person!

Extensions for Words and Phrases

Introducing Extensions

Frege's Principle

A Farewell to Psychol-ogism

Extensions for Words and Phrases

Truth Values as Extensions of Sentences

SO: unlike

- *current king of France* (whose extension is $\emptyset$),
- *the current king of France*

appears to have no extension.

Introducing Extensions

Frege's Principle

A Farewell to Psychol-ogism

Extensions for Words and Phrases

Truth Values as Extensions of Sentences

SO: unlike

- *current king of France* (whose extension is $\emptyset$),
- *the current king of France*

appears to have no extension. We will henceforth ignore such **void** descriptions. (Read chapter 9 for more on this ...)

Extensions for Words and Phrases

Introducing Extensions

Frege's Principle

A Farewell to Psychol-ogism

Extensions for Words and Phrases

Truth Values as Extensions of Sentences

Not alle nouns are count nouns — some are:

Extensions for Words and Phrases

Introducing Extensions

Frege's Principle

A Farewell to Psychol-ogism

Extensions for Words and Phrases

Truth Values as Extensions of Sentences

Not alle nouns are count nouns — some are:

- *mass nouns*: **milk**, **information**,...
  Hallmark: no plural (without meaning shift)

Extensions for Words and Phrases

Introducing Extensions

Frege's Principle

A Farewell to Psychol-ogism

Extensions for Words and Phrases

Truth Values as Extensions of Sentences

Not alle nouns are count nouns — some are:

- *mass nouns*: **milk**, **information**,...
  Hallmark: no plural (without meaning shift)

- *relational nouns*: **brother**, **copy**,...
  Hallmark: possessives receive "special" meaning

Extensions for Words and Phrases

Introducing Extensions

Frege's Principle

A Farewell to Psychol-ogism

Extensions for Words and Phrases

Truth Values as Extensions of Sentences

Not alle nouns are count nouns — some are:

- *mass nouns*: **milk**, **information**,...
  Hallmark: no plural (without meaning shift)
- *relational nouns*: **brother**, **copy**,...
  Hallmark: possessives receive "special" meaning
- *functional nouns*: **father**, **surface**,...
  Hallmark: relational plus inherent uniqueness

Extensions for Words and Phrases

Introducing Extensions

Frege's Principle

A Farewell to Psychol-ogism

Extensions for Words and Phrases

Truth Values as Extensions of Sentences

Not alle nouns are count nouns — some are:

- *mass nouns*: **milk**, **information**,...
  Hallmark: no plural (without meaning shift)
- *relational nouns*: **brother**, **copy**,...
  Hallmark: possessives receive "special" meaning
- *functional nouns*: **father**, **surface**,...
  Hallmark: relational plus inherent uniqueness

Mass nouns will be ignored in the following.

Extensions for Words and Phrases

Introducing Extensions

Frege's Principle

A Farewell to Psychol-ogism

Extensions for Words and Phrases

Truth Values as Extensions of Sentences

The extensions of relational and functional nouns can be identified with sets of **(ordered pairs)** of individuals.

Introducing Extensions
Frege's Principle
A Farewell to Psychol-ogism
Extensions for Words and Phrases
Truth Values as Extensions of Sentences

The extensions of relational and functional nouns can be identified with sets of **(ordered pairs)** of individuals.

Relational examples:

(4)

brother:
{⟨Ethan, Joel⟩, ⟨Joel, Ethan⟩, ⟨Deborah,Joel⟩, ⟨Deborah, Ethan⟩, . . . }

arm:
{⟨Ludwig, Ludwig's right arm⟩, ⟨Ludwig, Ludwig's left arm⟩, ⟨Paul, Paul's left arm⟩, . . . }

idea:
{⟨Albert, $E = mc^2$⟩, ⟨René, *COGITO*⟩, ⟨Bertie,$R \in R \Leftrightarrow R \notin R$⟩, . . . }

Extensions for Words and Phrases

Introducing Extensions

Frege's Principle

A Farewell to Psychologism

Extensions for Words and Phrases

Truth Values as Extensions of Sentences

Functional examples:

(5)

birthplace:
{⟨Adam, Paradise⟩, ⟨Eve, Paradise⟩, ⟨John, Liverpool⟩, ⟨Yoko, Tokyo⟩, . . . }

mother:
{⟨Cain, Eve⟩, ⟨Abel, Eve⟩, ⟨Stella, Linda⟩, ⟨Sean, Yoko⟩, . . . }

surface:
{⟨Mars, Mars's surface⟩, ⟨Earth, Earth's surface⟩, . . . }

Introducing Extensions
Frege's Principle
A Farewell to Psychol-ogism
Extensions for Words and Phrases
Truth Values as Extensions of Sentences

Functional examples:

(5)

birthplace:
{⟨Adam, Paradise⟩, ⟨Eve, Paradise⟩, ⟨John, Liverpool⟩, ⟨Yoko, Tokyo⟩, . . . }

mother:
{⟨Cain, Eve⟩, ⟨Abel, Eve⟩, ⟨Stella, Linda⟩, ⟨Sean, Yoko⟩, . . . }

surface:
{⟨Mars, Mars's surface⟩, ⟨Earth, Earth's surface⟩, . . . }

In addition to being relational, the extensions f of functional nouns in (5) are **functions**, i.e., they satisfy a **uniqueness** condition:

(6) If both $\langle a, v_1 \rangle \in f$ and $\langle a, v_2 \rangle \in f$, then $v_1 = v_2$.

Introducing Extensions
Frege's Principle
A Farewell to Psychol-ogism
Extensions for Words and Phrases
Truth Values as Extensions of Sentences

Functional examples:

(5)

birthplace:
{⟨Adam, Paradise⟩, ⟨Eve, Paradise⟩, ⟨John, Liverpool⟩, ⟨Yoko, Tokyo⟩, . . . }

mother:
{⟨Cain, Eve⟩, ⟨Abel, Eve⟩, ⟨Stella, Linda⟩, ⟨Sean, Yoko⟩, . . . }

surface:
{⟨Mars, Mars's surface⟩, ⟨Earth, Earth's surface⟩, . . . }

In addition to being relational, the extensions f of functional nouns in (5) are **functions**, i.e., they satisfy a **uniqueness** condition:

(6) If both $\langle a, v_1 \rangle \in f$ and $\langle a, v_2 \rangle \in f$, then $v_1 = v_2$.

The **extension** of a functional noun is a **function mapping** individuals to individuals.

Extensions for Words and Phrases

Introducing Extensions

Frege's Principle

A Farewell to Psychol-ogism

Extensions for Words and Phrases

Truth Values as Extensions of Sentences

Taking stock:

The extension of a **referential expression** — a name, a (non-void) definite description, a (referential) pronoun, etc. — is an **individual**, viz. its referent.

The extension of a **relational noun** is a **binary relation** among [= set of ordered pairs of] individuals.

The extension of a **functional noun** is a **function** mapping individuals to individuals.

Extensions for Words and Phrases

Introducing Extensions

Frege's Principle

A Farewell to Psychol-ogism

Extensions for Words and Phrases

Truth Values as Extensions of Sentences

Extensions of verbs and verb phrases

(7)

sleep: the set of sleepers

kiss: a relation between kissers and kissees, i.e., the set of pairs $\langle x, y \rangle$ such that x kisses y

donate: a **three-place relation**, a set of triples

Extensions for Words and Phrases

Introducing Extensions

Frege's Principle

A Farewell to Psychol-ogism

Extensions for Words and Phrases

Truth Values as Extensions of Sentences

(8)

type of expression	**type of extension**	**example**	**extension**
intransitive verb	set of individuals	*sleep*	the set of sleepers
transitive verb transitive verb	set of pairs of individuals	*eat*	the set of pairs ⟨eater, eaten⟩
ditransitive verb ditransitive verb	set of triples of individuals	*donate*	the set of triples ⟨donator, recipient, donation⟩

Truth Values as Extensions of Sentences

Introducing Extensions

Frege's Principle

A Farewell to Psychol-ogism

Extensions for Words and Phrases

Truth Values as Extensions of Sentences

(9) *Parallelism between valency and type of extension:*
The extension of an *n*-place verb is always a set of *n*-tuples.

Truth Values as Extensions of Sentences

(10) *The Pope shows the President the Vatican Palace*

(11)

verb or verb phrase	valency	extension
shows	3	the triples $\langle a, b, c \rangle$ where *a* shows *b* to *c*
shows the President	2	the pairs $\langle a, b \rangle$ where *a* shows *b* to the President
shows *the President* *the Vatican Palace*	1	the 1-tuples $\langle a \rangle$ where *a* shows the Vatican Palace to the President

Introducing Extensions

Frege's Principle

A Farewell to Psychologism

Extensions for Words and Phrases

Truth Values as Extensions of Sentences

Truth Values as Extensions of Sentences

Introducing Extensions
Frege's Principle
A Farewell to Psychol-ogism
Extensions for Words and Phrases
Truth Values as Extensions of Sentences

(10) *The Pope shows the President the Vatican Palace*

(11)

verb or verb phrase	valency	extension
shows	3	the triples $\langle a, b, c \rangle$ where *a* shows *b* to *c*
shows the President	2	the pairs $\langle a, b \rangle$ where *a* shows *b* to the President
shows *the President the Vatican Palace*	1	the 1-tuples $\langle a \rangle$ where *a* shows the Vatican Palace to the President

(12)

sentence	valency	extension
The Pope shows the President the Vatican Palace	0	the 0-tuples $\langle\rangle$ where the Pope shows the Vatican Palace to the president

Truth Values as Extensions of Sentences

Introducing Extensions
Frege's Principle
A Farewell to Psychol-ogism
Extensions for Words and Phrases
Truth Values as Extensions of Sentences

(13)

sentence	valency	extension
The Pope shows the President the Vatican Palace	0	the 0-tuples $\langle\rangle$ where the Pope shows the Vatican Palace to the president

Truth Values as Extensions of Sentences

(13)

sentence	valency	extension
The Pope shows the President the Vatican Palace	0	the 0-tuples $\langle\rangle$ where the Pope shows the Vatican Palace to the president

Standard Assumption 1

There is precisely one zero-tuple, viz., the empty set $\emptyset$.

Introducing Extensions
Frege's Principle
A Farewell to Psychol-ogism
Extensions for Words and Phrases
Truth Values as Extensions of Sentences

Truth Values as Extensions of Sentences

Introducing Extensions
Frege's Principle
A Farewell to Psychol-ogism
Extensions for Words and Phrases
Truth Values as Extensions of Sentences

(13)

sentence	valency	extension
The Pope shows the President the Vatican Palace	0	the 0-tuples $\langle\rangle$ where the Pope shows the Vatican Palace to the president

Standard Assumption 1

There is precisely one zero-tuple, viz., the empty set $\emptyset$.

Two cases:

- IF the Pope does NOT show the Vatican Palace to the president, then NO zero-tuple satisfies the condition that the Pope shows the Vatican Palace to the president

Truth Values as Extensions of Sentences

Introducing Extensions
Frege's Principle
A Farewell to Psychol-ogism
Extensions for Words and Phrases
Truth Values as Extensions of Sentences

(13)

sentence	valency	extension
The Pope shows the President the Vatican Palace	0	the 0-tuples $\langle\rangle$ where the Pope shows the Vatican Palace to the president

Standard Assumption 1

There is precisely one zero-tuple, viz., the empty set $\emptyset$.

Two cases:

- IF the Pope does NOT show the Vatican Palace to the president, then NO zero-tuple satisfies the condition that the Pope shows the Vatican Palace to the president and so the extension in (13) is empty

Truth Values as Extensions of Sentences

Introducing Extensions
Frege's Principle
A Farewell to Psychol-ogism
Extensions for Words and Phrases
Truth Values as Extensions of Sentences

(13)

sentence	valency	extension
The Pope shows the President the Vatican Palace	0	the 0-tuples $\langle\rangle$ where the Pope shows the Vatican Palace to the president

Standard Assumption 1

There is precisely one zero-tuple, viz., the empty set $\emptyset$.

Two cases:

- IF the Pope does NOT show the Vatican Palace to the president, then NO zero-tuple satisfies the condition that the Pope shows the Vatican Palace to the president and so the extension in (13) is empty, i.e.: $\emptyset$.
- IF the Pope DOES show the Vatican Palace to the president, then ANY zero-tuple satisfies the condition that the Pope shows the Vatican Palace to the president

Truth Values as Extensions of Sentences

Introducing Extensions
Frege's Principle
A Farewell to Psychol-ogism
Extensions for Words and Phrases
Truth Values as Extensions of Sentences

(13)

sentence	valency	extension
The Pope shows the President the Vatican Palace	0	the 0-tuples $\langle\rangle$ where the Pope shows the Vatican Palace to the president

Standard Assumption 1

There is precisely one zero-tuple, viz., the empty set $\emptyset$.

Two cases:

- IF the Pope does NOT show the Vatican Palace to the president, then NO zero-tuple satisfies the condition that the Pope shows the Vatican Palace to the president and so the extension in (13) is empty, i.e.: $\emptyset$.
- IF the Pope DOES show the Vatican Palace to the president, then ANY zero-tuple satisfies the condition that the Pope shows the Vatican Palace to the president and so the extension in (13) is the set of all 0-tuples

Truth Values as Extensions of Sentences

Introducing Extensions
Frege's Principle
A Farewell to Psychol-ogism
Extensions for Words and Phrases
Truth Values as Extensions of Sentences

(13)

sentence	valency	extension
The Pope shows the President the Vatican Palace	0	the 0-tuples $\langle\rangle$ where the Pope shows the Vatican Palace to the president

Standard Assumption 1

There is precisely one zero-tuple, viz., the empty set $\emptyset$.

Two cases:

- IF the Pope does NOT show the Vatican Palace to the president, then NO zero-tuple satisfies the condition that the Pope shows the Vatican Palace to the president and so the extension in (13) is empty, i.e.: $\emptyset$.
- IF the Pope DOES show the Vatican Palace to the president, then ANY zero-tuple satisfies the condition that the Pope shows the Vatican Palace to the president and so the extension in (13) is the set of all 0-tuples, i.e.: $\{\emptyset\}$.

Truth Values as Extensions of Sentences

Introducing Extensions

Frege's Principle

A Farewell to Psychologism

Extensions for Words and Phrases

Truth Values as Extensions of Sentences

Two cases:

- If the Pope does not show the Vatican Palace to the president, then the extension in (13) is: $\emptyset$.
- If the Pope does show the Vatican Palace to the president, then the extension in (13) is: $\{\emptyset\}$.

Truth Values as Extensions of Sentences

Introducing Extensions

Frege's Principle

A Farewell to Psychologism

Extensions for Words and Phrases

Truth Values as Extensions of Sentences

Two cases:

- If the Pope does not show the Vatican Palace to the president, then the extension in (13) is: $\emptyset$.
- If the Pope does show the Vatican Palace to the president, then the extension in (13) is: $\{\emptyset\}$.

(Wildly) generalizing:

- If a (declarative) sentence is false, its extension is: $\emptyset$.
- If a (declarative) sentence is true, its extension is: $\{\emptyset\}$.

Truth Values as Extensions of Sentences

Introducing Extensions

Frege's Principle

A Farewell to Psychologism

Extensions for Words and Phrases

Truth Values as Extensions of Sentences

Two cases:

- If the Pope does not show the Vatican Palace to the president, then the extension in (13) is: $\emptyset$.
- If the Pope does show the Vatican Palace to the president, then the extension in (13) is: $\{\emptyset\}$.

(Wildly) generalizing:

- If a (declarative) sentence is false, its extension is: $\emptyset$.
- If a (declarative) sentence is true, its extension is: $\{\emptyset\}$.

Truth Values as Extensions of Sentences

Introducing Extensions

Frege's Principle

A Farewell to Psychologism

Extensions for Words and Phrases

Truth Values as Extensions of Sentences

(Wildly) generalizing:

- If a (declarative) sentence is false, its extension is: $\emptyset$.
- If a (declarative) sentence is true, its extension is: $\{\emptyset\}$.

Standard Assumption 2

$\emptyset = 0, \{\emptyset\} = 1$.

Truth Values as Extensions of Sentences

Introducing Extensions

Frege's Principle

A Farewell to Psychologism

Extensions for Words and Phrases

Truth Values as Extensions of Sentences

(Wildly) generalizing:

- If a (declarative) sentence is false, its extension is: $\emptyset$.
- If a (declarative) sentence is true, its extension is: $\{\emptyset\}$.

Standard Assumption 2

$\emptyset = 0, \{\emptyset\} = 1$.

(14) *Frege's Generalization*
The extension of a sentence S is its truth value, i.e., 1 if S is true and 0 if S is false.