ESSLLI Summerschool 2014: Intro to Compositional Semantics

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Third Lecture: Composing Extensions

The set theoretic object that constitutes the reference of an expression in a particular situation *s* is called its extension in *s*.

If α is such an expression, its extension is denoted by $[\![\alpha]\!]_s$.

Given two expressions α and β forming a constituent $\boxed{\alpha\ \beta}$, what is $\left[\!\left[\boxed{\alpha\ \beta}\right]\!\right]_s$?

Given Frege's Principle, this must be a function *f* such that

(1) $$\left[\!\left[\boxed{\alpha\ \beta}\right]\!\right]_s = f([\![\alpha]\!]_s, [\![\beta]\!]_s)$$

But which function?

This depends on the nature of α and β, but also on the mode of syntactic combination.

We assume roughly 4 different modes of combination:

- functional application
- “plugging” or arity-reduction
- “predicate modification”
- set abstraction

Example 1: Functional nouns

Assume Berta is John's mother. Then:

(2) $[\![\boxed{\text{John's mother}}]\!]_s = [\![\text{mother}]\!]_s([\![\text{John}]\!]_s) = \text{Berta}$

General rule:

(3) $[\![\boxed{\text{term's functional noun}}]\!]_s = [\![\text{functional noun}]\!]_s([\![\text{term}]\!]_s)$

Convention: in mixed expressions that contain both meta-language and object language, the object language part is colored blue.

Terminology: by a term be mean any referential expression (proper name, definite description, pronoun, . . .)

Example 2: Truth tables

(4) Harry is reading or Mary is writing

(5)

$[\![\text{Harry is reading}]\!]_s$	$[\![\text{Mary is writing}]\!]_s$	$[\![(4)]\!]_s$
1	1	1
1	0	1
0	1	1
0	0	0

Accordingly:

$$[\![\text{or}]\!] = \{\langle\langle 1,1\rangle,1\rangle,\langle\langle 1,0\rangle,1\rangle,\langle\langle 0,1\rangle,1\rangle,\langle\langle 0,0\rangle,0\rangle\}$$

Compositional semantic rule:

(6) $\left[\!\left[\boxed{S_1 \text{ or } S_2} \right]\!\right]_s = [\![\text{or}]\!](\langle [\![S_1]\!]_s, [\![S_2]\!]_s \rangle)$

Example 3: Definite descriptions

Assume that in a certain situation s, the teacher in s is Harry. Then

(7) $\left[\!\left[\ \boxed{\text{the teacher}}\ \right]\!\right]_s = [\![\text{the}]\!]\,([\![\text{teacher}]\!]_s) = \text{Harry}$

For this to work we assume that *the* denotes a function. Which one? The function that assigns to a singleton set its only element (undefined for non-singletons):

(8) $[\![\text{the}]\!] = \{\langle X, y\rangle : X = \{y\}\}$

Compositional semantic rule:

(9) $\left[\!\left[\ \boxed{\text{the noun phrase}}\ \right]\!\right]_s = [\![\text{the}]\!]\,([\![\text{noun phrase}]\!]_s)$

Syntactic terminology: nouns are special noun phrases.

Plugging = arity-reduction:

(10) If R is an n-place relation (i.e. set of n-tuples $\in D_1 \times D_2 \times \ldots \times D_n$) and $y \in D_n,\ n \geq 1$, then
Right Edge Plugging (y is a plug for the last argument position):

$$R \overrightarrow{*} y := \{ \langle x_1, \ldots, x_{n-1} \rangle : \langle x_1, \ldots, x_{n-1}, y \rangle \in R \}$$

and Left Edge Plugging (y is a plug for the first argument position):

$$R \overleftarrow{*} y := \{ \langle x_2, \ldots, x_n \rangle : \langle y, x_2, \ldots, x_n \rangle \in R \}$$

We say that the last (first) argument position is plugged by y. The result is arity reduction, i.e. an $n-1$-place relation.

Recall that since $\langle x \rangle = x$, a 1-place relation is simply a set.

Notational conventions:

(11) In case R is a one-place relation, $R \overleftarrow{*} y$ and $R \overrightarrow{*} y$ coincide, both saying that $y \in R$; we then simply write $R * y$.

(12) Sometimes, the syntax of NL places a right edge plug on the left side of a predicate or relation; we then deliberately switch notation to $y \overrightarrow{*} R$ with the same meaning as $R \overrightarrow{*} y$. See below.

(13) [the pope [[shows [the V.P.]] to the president]]

(14) [der Papst [dem Präsidenten [[den Vatikanpalast] zeigt]]]

Compositional semantic rule:

(15) $[\![$ referential argument expression + relational expression $]\!]_s$

or $[\![$ relational expression + referential argument expression $]\!]_s$

$= [\![\text{relational expression}]\!]_s \overrightarrow{*} [\![\text{referential argument expression}]\!]_s$

Example:

(16) Referential argument expressions (= terms used as subjects or objects):

$[\![\text{der Papst}]\!]_s = [\![\text{the Pope}]\!]_s = p$

$[\![\text{dem Präsidenten}]\!]_s = [\![\text{the president}]\!]_s = o$

$[\![\text{den Vatikanpalast}]\!]_s = [\![\text{the V.P.}]\!]_s = v$

Relational expression:

$[\![\text{zeigt}]\!]_s = [\![\text{shows}]\!]_s = \{\langle p,o,v\rangle,\langle a,o,v\rangle,\langle a,b,c\rangle\}$

(17) Syntactic combinations:

$[\![\text{shows the V.P.}]\!]_s = \{\langle p,o,v\rangle,\langle a,o,v\rangle,\langle a,b,c\rangle\} \overrightarrow{*}\, v = \{\langle p,o\rangle,\langle a,o\rangle\}$

$[\![\boxed{\text{shows the V.P.}}\ \text{(to) the president}]\!]_s =$

$\{\langle p,o\rangle,\langle a,o\rangle\} \overrightarrow{*}\, o = \{\langle p\rangle,\langle a\rangle\} = \{p,a\} = R_1$

(17) $[\![\text{the Pope}\ \boxed{\text{shows the V.P. (to) the president}}]\!]_s = p * \{\langle p\rangle, \langle a\rangle\} = \{\langle p\rangle, \langle a\rangle\} * p = \{\langle p\rangle, \langle a\rangle\} \overrightarrow{*} p = \{\langle\,\rangle : \langle p\rangle \in \{\langle p\rangle, \langle a\rangle\}\} = \{\langle\,\rangle\} = 1$

(18) $[\![\text{John}\ \boxed{\text{shows the V.P. (to) the president}}]\!]_s = j * \{\langle p\rangle, \langle a\rangle\} = \{\langle\,\rangle : \langle j\rangle \in \{\langle p\rangle, \langle a\rangle\}$, hence (18) = $\{\ \} = \emptyset = 0$

Summary:

(17) = $[\![\text{the Pope}]\!]_s * [[[\![\text{shows}]\!]_s \overrightarrow{*} [\![\text{the V.P.}]\!]_s] \overrightarrow{*} [\![\text{the president}]\!]_s]$
= $[\![\text{the}]\!]([\![\text{Pope}]\!]_s) *$
$[[[\![\text{shows}]\!]_s \overrightarrow{*} [\![\text{the}]\!]([\![\text{V.P.}]\!]_s)] \overrightarrow{*} (\textit{to})\ [\![\text{the}]\!]([\![\text{president}]\!]_s)]$

(19) $[\![\text{der Papst dem Präsidenten den Vatikanpalast zeigt}]\!]_s =$

$[\![\text{der Papst}]\!]_s * [[\![\text{dem Präsidenten}]\!]_s \vec{*} [[\![\text{den V.P.}]\!]_s \vec{*} [\![\text{zeigt}]\!]_s]]]$

Note:
Since the subject and the objects in (19) are terms (ie. referential expressions denoting individuals), and given that the relation *zeigt* holds of/between individuals, we can use the notation $y \vec{*} R$ instead of $R \vec{*} y$ as defined in (12) above.

The notation thus reveals that semantic composition is the same in English and German.

(20) $\llbracket$the handsome boy from Berlin$\rrbracket_s$ =?

(21) [the [handsome [boy [from Berlin]]]]

(22) [the [[handsome boy] [from Berlin]]]

(23) $\llbracket$from Berlin$\rrbracket_s$ = $\llbracket$from$\rrbracket_s \overrightarrow{*}$ $\llbracket$Berlin$\rrbracket_s$

(24) General rule (*Predicate Modification*):

$\llbracket$noun phrase + modifying expression$\rrbracket_s$ =
$\llbracket$modifying expression + noun phrase$\rrbracket_s$ =
$\llbracket$noun phrase$\rrbracket_s \cap$ $\llbracket$modifying expression$\rrbracket_s$

(25) $[\![\text{the handsome boy from Berlin}]\!]_s$

a. $[\![\text{the}]\!]([\![\text{handsome}]\!]_s \cap [[\![\text{boy}]\!]_s \cap [[\![\text{from}]\!]_s \overset{\rightarrow}{*} [\![\text{Berlin}]\!]_s]])$

b. $[\![\text{the}]\!]([[\![\text{handsome}]\!]_s \cap [\![\text{boy}]\!]_s] \cap [[\![\text{from}]\!]_s \overset{\rightarrow}{*} [\![\text{Berlin}]\!]_s])$

$$A \cap (B \cap C)) = ((A \cap B) \cap C)$$

Cautionary notes:

- Some adjectives cannot be handled by Predicate Modification, ie. treated as intersective
- Some adjectives require in addition a standard of comparison

Assume

(26) $[\![\text{John is a murderer}]\!]_s = 1$ iff
John $\in [\![\text{murderer}]\!]_s$ iff
John $* [\![\text{murderer}]\!]_s$

In general:

(27) $[\![\text{term is a noun phrase}]\!]_s = [\![\text{term}]\!]_s * [\![\text{noun phrase}]\!]_s$

(28) $[\![\text{John is an alleged murderer}]\!]_s = 1$

incorrectly implies that John is a murderer (and that *John is alleged).

Rather, *alleged* should be analysed as a function from sets to sets, taking as argument the set of murderers and yielding the set of alleged murderers as value. As not all alleged murderers need to be murderers, on the contrary, this function is not intersective, it does not hold that $[\![\text{alleged}]\!]_s(M) \subseteq M$.

We then get an ambiguity caused by the scope of *alleged*:

(29) a. alleged [murderer from Berlin]
b. [alleged murderer] from Berlin

(30) a. $[\![\text{alleged}]\!]_s([\![\text{murderer}]\!]_s \cap [\![\text{from Berlin}]\!]_s)$
b. $[\![\text{alleged}]\!]_s([\![\text{murderer}]\!]_s) \cap [\![\text{from Berlin}]\!]_s$

Second problem:

(31) a. Jumbo is a small elephant
b. Jumbo is a big animal
c. Jumbo is big and small

Sounds like a contradiction. . .

Solution: Adjectives have an additional, syntactically not expressed argument:

(32) a. Jumbo is small (for an elephant)
b. Jumbo is big (for an animal)

The additional argument is a property X (*elephant, animal*, . . .) that has to be supplied pragmatically by the context of utterance. This property supplies the adjective with a standard of comparison.

(33) $[\![\text{small}_X]\!]_s = \{ y :$
y is small compared to the standard size of objects in $X \}$

Our fourth mode of operation, namely set formation (or comprehension in set theory) will become important at the level of LF. This will be discussed in the next chapter.